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The alphabetical notation system of Āryabhaṭa for numerals and its applications

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Abstract:
The wealth of ancient Indian Mathematical wisdom was so well developed that modern researchers are increasingly interested in discovering the knowledge of ancient India. The great mathematician and astronomer Āryabhaṭa I (476-540 AD) wrote Āryabhaṭīyam in 499 AD. The text is divided into four chapters (pādas): 1. Gītikāpāda 2. Gaṇitapāda 3. Kālakriyāpāda 4. Golapāda. The text mostly highlights the problems of Arithmetic, Algebra, Geometry, Trigonometry and Astronomy. In second verse of Gītikāpāda, alphabetical notation system for numerals is mentioned. Āryabhaṭa I has classified alphabets into two parts- I. Varga (group) letters; they have been divided into 5 groups. II. Avarga; they are not classified into Vargas. The varga letters ka to ma have value from 1, 2, 3, 4, 5. Up to 25 and the avarga letters ya to ha have value 30, 40, 50, 60….up to 100. This notation system has been applied by Āryabhaṭa I in his mathematical and astronomical calculations.
The research paper is aimed at determining astronomical and mathematical calculations through alphabetical notation system based on Sanskrit phonemes with examples & proofs from Āryabhaṭīyam and its commentaries.

Keyword: Aryabhata, Aryabhatiyam, gitikapada, varga, avarga letters

Introduction
In India, many systems of expressing numbers by means of words were developed in early ages. The numerals were expressed by names of things or beings. For example, the number one may be denoted by anything that is unique such as moon or earth. Similarly the number two may be denoted by any pair such as hands or eyes. The zero is denoted by words which mean void or sky. The numerals were largely used in mathematics, astronomy and other sciences. So, the need for a convenient method of expressing large numbers that occur so often in writing was strongly felt. The word numerals were invented to fulfil this need.
In most of the Sanskrit compositions, we find a tendency towards brevity in the text. There may be many reasons for this proclivity, for example, tradition of oral transfer of knowledge, lack of long-term writing resources, tradition of composing in strophes etc. The word system for numerals were discovered for these reasons also. Later, methods for expressing numbers with just alphabets were also developed. Panini used this kind of system in his work Aṣṭādhyāyī (500BC). Kaiyata explains that according to Panini, a=1, i=2 and u=3. Vararuchi (343AD-410AD) introduced the Katapayadi system for numeral notation in Chandravakya. In this system, a consonant followed by any vowel has a numerical value. In case of conjunction of two or more consonants, a consonant immediately followed by any vowel has only a numerical value. This Katapayadi system was extremely flexible.
Āryabhaṭa introduced an alphabetical notation system for numerals in his Āryabhaṭīya. This system is a very noticeable feature of Āryabhaṭīya. Āryabhaṭa is acknowledged as the greatest mathematician of Indian history. He was not just a mathematician but an astronomer too. His work in the realm of mathematics and astronomy is beyond the words of admiration. The work of Āryabhaṭa which is available to us is in the form of an excellent composition called Āryabhaṭīya. Āryabhaṭīya was written in 499 A.D. It consists of 121 stanzas which are beautifully composed in Gītikā and Āryā strophes. Āryabhaṭa himself quoted that he wrote Āryabhaṭīya in Kusumapura which is believed to be the other name of Pāṭliputra.
Āryabhaṭīya is divided into four pādas or chapters. These four chapters are:

1. Gītikāpāda
2. Gaṇitapāda
3. Kālakriyāpāda
4. Golapāda

Āryabhaṭīya deals with the problems of arithmetic, geometry, algebra, trigonometry and astronomy. The text deals in alphabetical numerical system, simple and quadratic equations, first degree indeterminate equations, table of trigonometrical calculations, natural numbers, rules of squares and cubes, value of pi etc.
There is a clear reference of number system in the Āryabhaṭīya. In Āryabhaṭīya, we find eka(1), daśa(10), śata(100), sahasra(1000), ayuta(10000), niyuta(100000), prayuta(1000000), koṭi(10000000), arbuda(100000000) and vṛnda(1000000000)in the following verse:

> ekam daś ca śatam ca sahasram tvyut niyute tathā prayutam.
> koṭyarbudam ca vṛndam sthānam daśguṇam syāt.
> (Āryabhaṭīya, Gaṇitapāda, verse 2)

Āryabhaṭa starts Āryabhaṭīya with the salutations to Brahma. After paying obeisance to Brahma, Āryabhaṭa explains the alphabetical notation system first and then proceeds to the other topics of subject. In the first chapter of Āryabhaṭīya, i.e., Gītikāpāda, the second verse mentions the alphabetical notation system for numerals. This verse is beautifully composed in Gītikā meter.
The verse is given below:

> vargākṣarāṇi varge’varge’vargākṣarāṇi kāt ṅmau yaḥ
> khadvinavake svarā nava varge’varge navāntyavarge vā.
> (Āryabhaṭīya, Gītikāpāda, verse 2)

The meaning of the above verse is as follows:
The varga letters are to be used in the varga places and the avarga letters in the avarga places. ‘y’ is equal to the sum of ‘ṅ’ and ‘m’. The nine vowels are to be used in the two nines of places varga and avarga. In order to understand this verse more explicitly, we need to look in the commentary of Āryabhaṭīya written by Bhaskarcharya I. Bhaskarcharya I explains the meaning of verse very clearly in his Dasagitika-sutravyakhya. In Sanskrit Alphabets, the letters ‘k’ to ‘m’ have been classified into five varga or groups. These 25 classified consonants from ‘k’ to ‘m’ are supposed to bear the numerical values from 1 to 25. The letters ‘y’ to ‘h’ are unclassified consonants. They have the following values:
y=ṅ+m
=5+25
=30
Similarly, r=40, l=50, v=60, ś=70, ṣ=80, s=90, h=100

Table 1: We can summarize these values of consonants in the following table

k 1	kh 2	g 3	gh 4	ṅ 5
c 6	ch 7	j 8	jh 9	ñ 10
ṭ 11	ṭh 12	ḍ 13	ḍh 14	ṇ 15
t 16	th 17	d 18	dh 19	n 20
p 21	ph 22	b 23	bh 24	m 25
y 30	r 40	l 50	v 60	
ś 70	ṣ 80	s 90	h 100	

The phrase ‘ṅmau yaḥ’ serves two purposes. First, it gives the value of alphabet ‘y’, i.e., 30. Secondly it suggests that the conjoint letter ṅm means ṅ+m. Using this idea, the system gives the nomenclature to the large numbers. The notational places are also classified into varga and avarga places. From right to left, the odd places, like units place, hundreds place etc. are known as varga places and the even places such as tens place, thousands place are known as avarga places. The notational places from right to left are denoted by the nine vowels as follows:

au au ai ai o o e e ḷ ḷ ṛ ṛ u u i i a a
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

The difference between short and long vowel is not considered and both bear the same value. While representing a number, consonant denotes a number and the vowel denotes the place where the number is to be located.
For example, let us find the numerical value of gi:

A	V	A	V	A	V
u	u	i	i	a	a
0	0	0	g 3	0	0

Hence, gi=300
Similarly, numerical value of makhi:

A	V	A	V	A	V
u	u	i	i	a	a
0	0	0	kh 2	m 25	0

Hence, we calculate the value of makhi=225
In order to mention all the large numbers, Āryabhaṭa used this alphabetical notation system. Some of the applications of this alphabetical notation system are discussed here:
Āryabhaṭa applied this system for writing the numerals in the following way:

> kāhomanavo dḥ manuyuga śkha gatāste ca manuyuga chnā ca.
> kalpāderyugapādā ga ca gurudivasācca bharatāt pūrvam.
> (Āryabhaṭīya, Gītikāpāda, verse 5)

Meaning: A day of Brahmā or a kalpa is equal to 14(dḥa) manus. A manu is equal to the time period of 72(śkha) yugas. Since the beginning of this kalpa up to the Thursday of

Bharata or Mahabharata, 6(ca) manus, 27(chnā) yugas and 3(ga) yugapada or 3 quarters of yuga have elapsed.

- dḥa :

A	V	A	V
i	i	a	a
0	0	0	dḥ
			14

dḥa = 14.

- śkha :

A	V	A	V
i	i	a	a
0	0	ś	kh
0	0	7	2

śkha = 72.

- ca :

A	V	A	V
i	i	a	a
0	0	0	c
0	0	0	6

ca = 6

- chnā :

A	V	A	V
i	i	a	a
0	0	0	ch=7
			n= 20
			20+7=27
			chnā = 27

- ga :

A	V	A	V
i	i	a	a
0	0	0	g
0	0	0	3

ga = 3

The number of eastward revolutions of Sun, Moon, Earth and other planets in a mahayuga are given by the following verse using the alphabetical notation system.

> yugaravibhagaṇāḥ khyughṛ śaśi cayagiyiṅuśuchlṛ ku ṅi śibuṇḷṣkhṛ prāk.
> śani dḥuṅvighva guru khricyubha kuja bhadlijhnukhṛ bhṛgubudhasaurāḥ.
> (Āryabhaṭīya, Gītikāpāda, verse 3)

Meaning: There are 4320000(khyughṛ) eastward revolutions of the Sun, 57753336(cayagiyiṅuśuchlṛ) revolutions of the Moon, 1582237500(ṅiśibuṇḷṣkhṛ) revolutions of the earth, 146564 (dḥuṅvighva) revolutions of the Saturn, 364224 (khricyubha) revolutions of the Jupiter, 2296824(bhadlijhnukhṛ) revolutions of the Mars in a yuga. The revolutions of Mercury and Venus are same as those of the Sun.

Sun's revolutions = khyughṛ

A	V	A	V	A	V	A	V
ṛ	ṛ	u	u	i	i	a	a
0	gh	y	kh	0	0	0	0
0	4	3	2	0	0	0	0

khyughṛ = 4320000

Moon's revolutions = cayagiyiṅuśuchlṛ

A	V	A	V	A	V	A	V
ṛ	ṛ	u	u	i	i	a	a
l	ch	ś	ṅ	y	g	y	c
5	7	7	5	3	3	3	6

cayagiyiṅuśuchlṛ = 57753336

Earth's revolutions = ṅiśibuṇḷṣkhṛ

A	V	A	V	A	V	A	V	A	V
ḷ	ḷ	ṛ	ṛ	u	u	i	i	a	a
0	ṅ	ṣ	kh	0	b	ś	ṅ	0	0
1	5	8	2	2	3	7	5	0	0

ṅiśibuṅḷṣkhṛ = 1582237500

Saturn's revolutions = dḥuṅvighva

A	V	A	V	A	V
u	u	i	i	a	a
0	dḥ	v	ṅ	v	gh
1	4	6	5	6	4

dḥuṅvighva = 146564

Jupiter's revolutions = khricyubha

A	V	A	V	A	V
u	u	i	i	a	a
y	c	r	kh	0	bh
3	6	4	2	2	4

khricyubha = 364224

Thus, we see that the knowledge of using alphabetical notation system given by Āryabhaṭa is necessary to understand the mathematical and astronomical calculations propounded by him in the text Āryabhaṭīya. Āryabhaṭa acquired the knowledge of number system from Vedas and expanded that knowledge by such kind of notation system for numbers and numerals. This alphabetical notation system is so precise that each and every consonant and vowel contributes in extracting the exact numerical value and we cannot omit or increase a single alphabet. The omission or extension of a single letter can change the entire numerical value. Therefore, we can say that Āryabhaṭa developed, explored and utilised his alphabetical notation system with a great scientific approach.

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