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Mentions of Dhruva in Indian Astronomical Treatises – a Possible Pole Star?

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Abstract
The pole star is one of the most prominent features of the night sky. It is known that the star that occupies the pole changes over the centuries owing to the precession of equinoxes. Despite this, several texts in the Indian tradition mention in clear words a pole star, usually called dhruva (which means 'fixed') or dhruvatārā (tārā meaning 'star'). Researchers have proposed candidate stars for mentions in the Vedas and Purāṇas in the past. However, a large number of mentions, which are purely astronomical in nature, are also found in the siddhāntas of Indian astronomers. They have not been discussed comprehensively until now. This work presents these together and discusses the implications of these mentions.

Keyword: Dhruva, siddhānta, pole star, north pole

Introduction
The pole star is one of the preeminent stars in the night sky. As it appears fixed in the night sky, the pole star has become a common metaphor for anything changeless and represents continuity and stability in several cultures. Its Sanskrit name, 'dhruva', meaning changeless or fixed, illustrates well this connotation. Even today, the word dhruva or dhruvatārā in Sanskrit and Indian languages, and the expressions "pole star" or "north star" in English are often used to denote something that is changeless or fixed, and serving as a guide for others, as this star can readily be used by travellers who wish to know their location and decide where they wish to go.

As with other celestial objects, various stories and legends are associated with the pole star, a prominent and famous one being that of Dhruva, the prince, who won the favour of Lord Viṣṇu, as stated in the Purāṇas (for example, in the Bhāgavata Purāṇa).

From an astronomical point of view, we know that the earth rotates about its axis every day and causes the sun, moon, stars and planets to move across the sky from east to west. If the axis of the earth's rotation is extended to two points in the sky – one in the north and the other in the south, all celestial objects will appear to move around these points. These points are called the celestial poles. Any object located at those points does not appear to move. A pole star is such a star located at one of these poles, and usually applies to the star located at the north pole.

A popular conception today is that the pole star remains fixed in the sky. However, astronomy tells us that over the course of centuries, the north celestial pole changes its position. At any given point in time, it may point towards a star, or near one, such that that star becomes the pole star. However, eventually, the north celestial pole shifts away from this point.

Due to the precession of equinoxes (also called axial precession), the earth's axis changes position. It is this change that causes the location of the pole to shift continually. Currently, this occurs at the rate of about $50.3''$ per year. This change was recognized by several ancient societies including ancient Indian astronomers who called it ayanacalana.

The precession of equinoxes implies that no star can remain at or near the pole for very long as it keeps shifting. The current pole star is Polaris. This star came near the pole only in the past few centuries. However, references to the dhruva in the Vedas, Purāṇas, astronomical and other literature, such as in the story highlighted above predate this event. Hence, it becomes necessary to explore the nature and identity of the star mentioned in these texts.

Some work has been done in this regard in the past: based on various factors including the appearance of the constellation created by nearby stars, various previous authors such as Jacobi (1894) [19] and Iyengar (2011) identify the dhruva mentioned in the Vedic and Purāṇic literature as Thuban (α-Draconis).

The references to dhruva among Indian astronomers (c. 500 – 1700 CE), however, have not been looked at in detail so far. Some such as Jacobi [1] have even denied that such mentions exist, in contrast to what is found in the texts. The primary purpose of this work is to bring out these references, present them together and discuss their implications.

Mentions of Dhruva

Siddhāntas are treatises that cover mathematical astronomy. The oldest datable siddhānta currently available that is preserved in its entirety, is Āryabhaṭīya of Āryabhaṭa, dated to 499 CE. It is believed that there were siddhāntas prior to as well. However, they are no longer available. The period of Indian astronomy when siddhāntas have been in vogue is usually called the Siddhāntic period. This period essentially extends from the first known siddhāntas in the fifth century CE and probably a few centuries before that up until the colonial era of Indian history.

During the Siddhāntic period of Indian astronomy, it is observed that in astronomical literature, the word dhruva often indicated the north celestial pole, since it remains fixed ('dhruva'). There are however several astronomers who mention the presence of a pole star in their works. References to dhruva from these texts are now presented in a chronological order.

Varāhamihira

The Pañcasiddhāntikā of Varāhamihira (sixth century CE) is an important text in Indian astronomy, since it is one of the oldest available works in the subject and as it summarises and provides a comparison of five ancient siddhāntas which are now lost. Different chapters and sections in the work cover different siddhāntas or aspects thereof.

The relevant chapter in this context is the thirteenth one, which is titled "Trailokyasaṁsthāna", which is not directly linked with any particular siddhānta within Varāhamihira's work.

The following verse (Pañcasiddhāntikā 13.26) deals with the location of the pole star in the sky when viewed from different places on earth:

> लङ्कास्था भूलग्नां नभसो मध्यस्थितां च मेरुगताः ।
> ध्रुवतारामीक्षन्ते तदन्तरालेऽन्तरोपगताः ।।
> laṅkāsthā bhūlagnāṃ nabhaso madhyasthitāṃ ca merugatāḥ |
> dhruvatārāmīkṣante tadantarāle'ntaropagatāḥ ||
>
> Those present at the equator (laṅkā) observe the pole star as clinging to the earth (i.e., at the horizon). Those at the (north) pole, see it at the zenith. Those in between, see the pole star in the middle (i.e., with an altitude greater than 0° but less than 90°).

Since Varāhamihira says "dhruvatārām īkṣante" ("see the pole star"), it may be deduced that this is a star(tārā) that he feels was visible.

The verse highlights three cases regarding the observer:

1. If the observer is at the equator, the pole star will be seen at the horizon.
2. If the observer is at the north pole, the pole star will be seen at the zenith.
3. If the observer is at a latitude between 0° and 90°, the pole star is visible between the horizon and the zenith.

For those located between the equator and the pole on earth, the pole star is located somewhere in the middle. For finding the latitude from the altitude of the star, Varāhamihira provides the following verses (Pañcasiddhāntikā 13.30-33):

> सलिलेन समं कृत्वा तुङ्गं फलकं यथादिशं दृष्ट्वा ।
> दक्षिणकोट्यां शङ्कुं फलकप्रमितं व्यवस्थाप्य ।।
> ऋजुशङ्कुबुध्नविन्यस्तलोचनो नामयेत्तथा शङ्कुम् ।
> भवति यथा शङ्क्वग्रं ध्रुवतारादृष्टिमध्यस्थम् ।।
> पतितेन भवति वेधो लङ्कायामूर्ध्वगेन तु सुमेरौ ।
> विनतेन चान्तराले फलके चाक्षोर्ध्वसूत्रसमम् ।।
> तत्रावलम्बको यः सोऽक्षज्या तस्य शङ्कुविवरं यत् ।
> विषुवदवलम्बकोऽसौ याम्योत्तरदिक्प्रसिद्धिकरः ।।
> salilena samaṃ kṛtvā tuṅgaṃ phalakaṃ yathādiśaṃ dṛṣṭvā,
> dakṣiṇakoṭyāṃ śaṅkuṃ phalakapramitaṃ vyavasthāpya.
> ṛjuśaṅkubudhnavinyastalocano nāmayettathā śaṅkum,
> bhavati yathā śaṅkvagraṃ dhruvatārādṛṣṭimadhyastham.
> patitena bhavati vedho laṅkāyāmūrdhvagena tu sumerau,
> vinatena cāntarāle phalake cākṣordhvasūtrasamam.
> tatrāvalambako yaḥ so'kṣajyā tasya śaṅkuvivaraṃ yat,
> viṣuvadavalambako'sau yāmyottaradikprasiddhikaraḥ.
>
> An elevated board must be placed in alignment with the directions, on a place that has been confirmed to be flat with water. On the southern end, a śaṅku must be affixed such that its size (length) is the same as that of the board (in a north-south direction). It must then be lowered so that the pole star is observed at the end of the śaṅku. If it falls (i.e., is lowered completely), (the observer) is at Laṅkā (the equator) and if it points upwards, (he is) at Sumeru (the north pole). The perpendicular dropped (from the tip of the śaṅku onto the board) is the R-sine of the latitude. The difference between the perpendicular and the gnomon amounts tothe R-cosine of the latitude. The R-cosine line follows the north-south direction.

To measure the altitude of dhruva, Varāhamihira states that a board must be placed on a flat place. Whether the place chosen is flat or not is determined by pouring water on it. If the water flows to any one direction, there is a slope and that place cannot be chosen. Once the appropriate place is chosen or the desired surface brought in to the flat shape, the board should be placed in such a way that its edges are aligned to the cardinal directions.

Śaṅku refers to a gnomon. In the Indian context, this comprises of a thin cylindrical pole of a fixed length. Here, however, instead of fixing the śaṅku onto the ground, as is usually specified in Indian astronomical treatises, Varāhamihira essentially asks the observer to attach it onto the southern end of the board, such that it can be tilted as required. The length of the board along the north-south direction and the height of the śaṅku must be the same. After this, at night time, the observer must tilt the śaṅku such that its end points in the direction of the pole star.

Clarifying his previous point, he states that if the śaṅku is lowered completely, the observer is at the equator, as would be expected. If it is not lowered at all, the pole star is at the zenith and the observer is at the north pole of the earth.

[1] Jacobi (1894: 158) states "… the Indian astronomers do not name a pole star"

A possible error can occur in this reading if the śaṅku is not lowered exactly. However, if lowered to the correct angle, the angle by which the śaṅku is raised from the board corresponds to the altitude of the pole star in the sky, which is the same as the latitude. Hence if a perpendicular is dropped from the tip of the śaṅku to the board, its height would correspond to the R-sine[2] of the latitude (also called akṣajyā). The distance from the southern end of the board, upto the point where the perpendicular meets the board is the R-cosine of the latitude (also called lambajyā). This will be aligned with the north-south line.

Varāhamihira's observations of dhruva are intended to measure latitude and ascertain the directions. However, they appear to assume the presence of a visible pole star.

Sūryasiddhānta

The present Sūryasiddhānta has not been confidently dated. However, Varāhamihira mentions another Sūryasiddhānta in his Pañcasiddhāntikā. Since the parameters of the two systems are different, it is believed that the present text postdates Varāhamihira.

The Sūryasiddhānta mentions dhruva in a few places. One of them (12.73) is:

> भचक्रं ध्रुवयोर्नद्धमाक्षिप्तं प्रवहानिलः ।
> पर्येत्यजस्रं तन्नद्धा ग्रहकक्षा यथाक्रमम् ।।
> bhacakraṃ dhruvayornaddhamākṣiptaṃ pravahānilaḥ,
> paryetyajasraṃ tannaddhā grahakakṣā yathākramam.
>
> The circle of asterisms, bound at the two poles, impelled by the provector (pravaha) winds, revolves eternally; attached to that are the orbits of the planets, in their order. (Burgess, 1935: 292)

This dhruva may be interpreted as the celestial pole or the pole star, hence this reference is ambiguous. However, the following two verses (12.43-44) mention the pole star clearly:

> मेरोरुभयतो मध्ये ध्रुवतारे नभःस्थिते ।
> निरक्षदेशसंस्थानमुभये क्षितिजाश्रये ।।
> अतो नाक्षत्रोच्छ्रयस्तासु ध्रुवयोः क्षितिजस्थयोः ।
> नवतिर्लम्बकांशास्तु मेरावक्षांशकास्तथा ।।
> merorubhayato madhye dhruvatāre nabhaḥsthite,
> nirakṣadeśasaṃsthānamubhaye kṣitijāśraye.
> ato nākṣatrocchrayastāsu dhruvayoḥ kṣitijasthayoḥ,
> navatirlambakāṃśāstu merāvakṣāṃśakāstathā.
>
> In both directions from Meru are two pole-stars (dhruva-tārā), fixed in the midst of the sky; to those who are situated in places of no latitude (nirakṣa), both these have their place in the horizon. Hence, there is in those cities, no elevation of the pole, the pole-stars being situated in their horizon; but their degrees of co-latitude (lambaka) are ninety: at Meru, the degrees of latitude (akṣa) are of the same number. (Burgess, 1935: 286)

The pole star is clearly referred to as such. Not only does the text suppose a northern pole star but also a southern one.

At the equator (place of no latitude, in the text above), the two pole stars are located at the horizon. The elevation of the pole has also been referred to. This is dealt with in another verse (12.72) as well:

> ध्रुवोन्नतिर्भचक्रस्य नतिर्मेरुं प्रयास्यतः ।
> निरक्षाभिमुखं यातुर्विपरीते नतोन्नते ।।
> dhruvonnatirbhacakrasya natirmeruṃ prayāsyataḥ,
> nirakṣābhimukhaṃ yāturviparīte natonnate.
>
> To one going toward Meru, there take place an elevation of the pole (dhruva) and a depression of the circle of asterisms; to one going towards the place of no latitude, on the contrary, a depression of the former and an elevation of the latter. (Burgess, 1935: 291)

Based on these, it appears that the current Sūryasiddhānta is acquainted with a visible dhruva and measurement of its elevation.

Brahmagupta

At the outset of his treatise, Brāhmasphuṭasiddhānta (1966, 1.3), Brahmagupta states:

> ध्रुवताराप्रतिबद्धज्योतिश्चक्रं प्रदक्षिणगमादौ ।
> पौष्णाश्विन्यन्तस्थैः सह ग्रहैर्ब्रह्मणा सृष्टम् ।।
> dhruvatārāpratibaddhajyotiścakraṃ pradakṣiṇagamādau |
> pauṣṇāśvinyantasthaiḥ saha grahairbrahmaṇā sṛṣṭam ||
>
> The circle of luminaries (jyotiścakra), attached to the [two] pole stars, moving clockwise, with Pauṣṇa (Revatī) and Aśvinī at its ends, was first created, along with the planets (graha), by Brahmā.

This is a clear reference to the pole star as he uses the word 'dhruvatārā'. Brahmagupta refers to the dhruva, without specifying 'tārā', in the following verses (21.3b-5):

> खे भगणाक्षाग्रस्थावुपर्यधश्च ध्रुवौ तेषां ।।
> ध्रुवयोर्बद्धं सव्यगममराणां क्षितिजसंस्थमुडुचक्रम् ।
> अपसव्यगमसुराणां भ्रमति प्रवहानिलाक्षिप्तम् ।।
> अन्यत्र सर्वतो दिशमुन्नमति भपञ्जरो ध्रुवो नमति ।
> लंकायामुडुचक्रं पूर्वापरगं ध्रुवौ क्षितिजे ।।
> khe bhagaṇākṣāgrasthāvuparyadhaśca dhruvau teṣāṃ.
> dhruvayorbaddhaṃ savyagamamarāṇāṃ
> kṣitijasaṃsthamuḍucakram,
> apasavyagamasurāṇāṃ bhramati pravahānilākṣiptam.
> anyatra sarvato diśamunnamati bhapañjaro dhruvo namati,
> laṅkāyāmuḍucakraṃ pūrvāparagaṃ dhruvau kṣitije.
>
> There are for them (the residents of the terrestrial north and south poles) two poles [located] in the sky, above and below [the extension of] the axis of the earth. Affixed onto the poles, the stellar sphere moves propelled by the pravaha (provector) wind, clockwise, for the gods (residents of the north pole), along the horizon, while it moves anticlockwise for the demons (residents of the south pole). In all other places, the stellar sphere rises while the pole diminishes (in altitude). In Laṅkā (the equator), the stellar sphere moves east to west with the poles being located at the horizon.

Bhāskara I – Someśvara

Bhāskara I (fl. 629 CE) authored a commentary on the Āryabhaṭīya of Āryabhaṭīya. Portions of the Golapāda in this are not available now, however, Someśvara (10th – 12th century CE) prepared a summary to Bhāskara's commentary. Two references to dhruva are found in this. The first occurs in the context of discussing the size of Meru stated by Āryabhaṭa (Gola 11) and a comparison of this with the ideas of the Purāṇas:

[2] Indian authors use a sine table where $\sin 90° \neq 1$, hence the word R-sine is used to express the sines taken in a circle which is not a unit circle.

... तस्माद्ध्रुवोन्नत्या आनीतमेव भुवः प्रमाणं सिद्धम् । ...
… tasmāddhruvonnatyā [ānītameva] bhuvaḥ pramāṇaṃ siddham. …

Hence, the size of the earth obtained from dhruvonnati (elevation of dhruva) is the one that is established (/correct).

The second reference occurs as a commentary to Āryabhaṭa's elaboration of the midnight sun (Gola 16). Someśvara states:

देशान्तरव्यवधानात् अन्यथा भचक्रार्धदर्शनं भवति । कश्चित् पुरुषः उत्तरेण गतः देशान्तरमेति तथात्वे ध्रुवमुपरि आरोहितं पश्यति, क्रमेण मेरुं प्राप्तस्य उपरि ध्रुवः भवति ।
deśāntaravyavadhānāt anyathā bhacakrārdhadarśanaṃ bhavati. kaścit puruṣaḥ uttareṇa gataḥ deśāntarameti tathātve dhruvamupari ārohitaṃ paśyati, krameṇa meruṃ prāptasya upari dhruvaḥ bhavati.

If a person goes to another place (apart from the north and south poles), the half of the celestial sphere that he observes is different. A person who goes northwards sees the dhruva risen in the sky (i.e., with non-zero altitude). When he eventually reaches Meru (the North Pole), the dhruva is up (i.e., at the zenith).

This explanation pertains to the situation of an observer who travels northwards from the equator up to the pole. Although the word 'tārā' has not been explicitly used, the text expects the observer to "see" the pole (paśyati meaning 'sees'). Thus, it appears that a visible star is indicated by his statements rather than the celestial pole.

Lalla

Lalla has also mentioned dhruva in the context of finding direction. The following verse segment is from his Śiṣyadhīvṛddhidatantra (4.2):

यत्र ध्रुवो धनपतेर्दिगसौ भवेद्वा
yatra dhruvo dhanapaterdigasau bhavedvā

… north, that is, in the direction of the polar star. (Chatterjee, 1981: 59-64)

This essentially associates the northern direction with dhruva, translated with "polar star" above.

Lalla criticises the viewpoint of the Jainas who state that there are two suns and two moons. To do this, he follows Brahmagupta by referring to the dhruvamatsya, a constellation of stars in the region of the north pole. Lalla states (20.44):

द्वितयं दिनरात्रिनाथयोः कथमेकान्तरितं तदुद्वजेत् ।
यदि किं ध्रुवतारकातिमेर्दिवसेनैव भवेत्परिभ्रमः ।।
dvitayaṃ dinarātrināthayoḥ kathamekāntaritaṃ tadudvrajet.
yadi kiṃ dhruvatārakātimerdivasenaiva bhavetparibhramaḥ.

How can the there be two suns and moons appearing one after another, if the fish of polar star(s) completes a revolution in one day itself?

To refer to the dhruvamatsya, Lalla uses the word 'dhruvatārakātimi' – dhruvatārakā refers to the 'pole star' and timi refers to a 'fish' or 'whale'. Chatterjee (1981) translates dhruvatārakātimi as 'circumpolar constellations.'

Vaṭeśvara

Vaṭeśvara states the following in Vaṭeśvara-siddhānta (3.4):

वृत्तं रवौ प्रविष्टे सममण्डलसंज्ञितं प्रभा या स्यात् ।
समपूर्वापरगा सा सौम्या यत्र ध्रुवः सा स्यात् ।।
vṛttaṃ ravau praviṣṭe samamaṇḍalasaṃjñitaṃ prabhā yā syāt,
samapūrvāparagā sā saumyā yatra dhruvaḥ sā syāt.

When the sun enters the circle called the prime vertical, the shadow (of a vertical gnomon) falls exactly east to west. Towards the north-pole, lies the north direction. (Shukla, 1985: 278)

This description thus serves to find direction. The word 'dhruva' is used to indicate north pole.

He also states the following for finding latitude (3.26):

शङ्कुं परिकल्प्य भुजं त्रिभुजेन विलोकयेद्ध्रुवमुदीच्याम् ।
यन्त्रेण दृष्टिभुजयोरविवराग्रा वा पलच्छाया ।।
śaṅkuṃ parikalpya bhujaṃ tribhujena vilokayeddhruvamudīcyām,
yantreṇa dṛṣṭibhujayoravivarāgrā vā palacchāyā.

One should observe the Pole Star towards the north along the hypotenuse of the triangle-instrument, assuming its base to be equal to the gnomon; then the upright (of the triangle-instrument), which lies between the line of vision and the base, will be equal to the equinoctial midday shadow. (Shukla, 1985: 288)

Vaṭeśvara also places the pole star in the context of nearby stars (8.16):

तिम्याकृतिताराणां ध्रुवतारा तनुतरा मध्ये ।।
timyākṛtitārāṇāṃ dhruvatārā tanutarā madhye. [3]

The pole star is the faint (tanu) star located in the middle (madhya) of the stars that are shaped as a timi (whale or fish).

As with Lalla, Vaṭeśvara uses the word 'timi' and refers to the dhruvamatsya. However, he mentions the pole star as a faint star located within this fish. Shukla (1985: 596) has interpreted the group of stars which is the polar fish or dhruvamatsya as the constellation Ursa Minor. This is a clear reference to a faint pole star.

Śrīpati

Śrīpati (c. 1050 CE) refers to a method of measuring latitude that involves finding the elevation of dhruva. The verses in which this occurs (Siddhāntaśekhara 4.122-123) are given below with a translation:

अम्भोभिः सुसमीकृते त्रिफलके दृष्ट्युच्छ्रिते स्थापये–
च्छङ्कुं तत्परिमाणमुत्तरदिशं कोटीं दृशं तत्तले ।
शङ्क्वग्रे भवति ध्रुवो यदि तदा मेरावुदक्कोटिके
लङ्कायां वसतिस्ततोऽन्यविषये तच्छङ्कुना भेदयेत् ।।
शङ्क्वग्रतो यत्र च सूत्रपातस्तच्छङ्कुमूलान्तरमत्र लम्बः ।
शङ्क्वग्रभूम्यन्तरमक्ष उक्तस्त्रिज्याग्रतस्तौ भवतोऽनुपातात् ।।
ambhobhiḥ susamīkṛte triphalake dṛṣṭyucchrite sthāpaye-
cchaṅkuṃ tatparimāṇamuttaradiśaṃ koṭīṃ dṛśaṃ tattale |

[3] Printed editions of this verse show the word 'tanutārā' in place of 'tanutarā'. However, considering the meter of this verse, the reading 'tanutarā' seems more appropriate.

śaṅkvagre bhavati dhruvo yadi tadā merāvudakkoṭike
laṅkāyāṃ vasatistato'nyaviṣaye tacchaṅkunā bhedayet ||
śaṅkvagrato yatra ca sūtrapātastacchaṅkumūlāntaramatra lambaḥ |
śaṅkvagrabhūmyantaramakṣa uktastrijyāgratastau bhavato'nupātāt ||

A gnomon must be placed on a board (triphalaka) that is at the height of the eyes and levelled well with water. The eyes are placed at the base of the board. If the pole is at the tip of the gnomon, [the observer is] at Meru (the North Pole). If it is at the base of the northern side, then [he lives] at Laṅkā (the Equator). If it is some other place, the pole must be measured using the gnomon. The distance between the point where the thread tied to the upper edge of the gnomon meets [the earth] and the base of the gnomon, is the lamba (cosine of the latitude), and the distance between the tip of the gnomon and the earth is the akṣa (sine of the latitude) by ratio.

The standard Indian gnomon is of the shape of a thin cylinder that is completely vertical. Śrīpati essentially uses a gnomon to measure the angles involved. The eyes observe from the level of the base of the gnomon, where a board is placed. The observer must ensure that the board is perfectly horizontal by holding water on it.
As in the case of Varāhamihira, the cases of the observer being at the pole, equator or in between have been indicated in the verse. Unlike Varāhamihira, however, Śrīpati uses the word dhruva and not dhruvatārā, to refer to the pole star. Given that he refers to locating the object in the sky, it seems to indicate that the object must have been bright and therefore the pole star.

Bhāskara II

Bhāskara II in his work Siddhāntaśiromaṇi, which is one of the most popular works on Indian astronomy, has also mentioned the dhruva at the outset of his work as follows (Kālamāna 13):

सृष्ट्वा भचक्रं कमलोद्भवेन ग्रहैः सहैतद्भगणादिसंस्थैः ।
शश्वद्भ्रमे विश्वसृजा नियुक्तं तदन्ततारे च तथा ध्रुवत्वे ।।
sṛṣṭvā bhacakraṃ kamalodbhavena grahaiḥ sahaitadbhagaṇādisaṃsthaiḥ,
śaśvadbhrame viśvasṛjā niyuktaṃ tadantatāre ca tathā dhruvatve.

The Creator having created the stellar circle along with the planets, placed the latter at the beginning of the circle, put them in constant revolution, at the same time putting the extreme two stars (on either side) in a fixed position. (Somayaji, 1980: 4)

He explains this in his auto-commentary (Vāsanābhāṣya) to the above verse as follows:

तथा तस्य भपञ्जरस्य यौ दक्षिणोत्तरावन्तौ तत्र ये तारे ते ध्रुवत्वे नियुक्ते ।।
tathā tasya bhapañjarasya yau dakṣiṇottarāvantau tatra ye tāre te dhruvatve niyukte.

Two stars, are placed, one at the north pole and the other at the south, and they are fixed. (Somayaji, 1980: 5)

The word tāre (dual of tārā) makes it clear that Bhāskara II intends that there are stars at the pole. He also states in his auto-commentary, immediately prior to Bhuvanakośa 51, "atastatra dhruvarkṣasaṁsthānamāha". Usage of the word dhruvarkṣa meaning pole star appears to substantiate this.
He also states (Tripraśnavāsanā 33):

यन्त्रवेधविधिना ध्रुवोन्नतिर्या नतिश्च भवतोऽक्षलम्बकौ ।
तौ क्रमाद्विषुवदह्न्यहर्दले येऽथवा नतलमुन्नता लवाः ।।
yantravedhavidhinā dhruvonnatiryā natiśca bhavato'kṣalambakau,
tau kramādviṣuvadahnyahardale ye'thavā natalamunnatā lavāḥ.

The altitude of the polar star and the zenith distance, as found by observation, give respectively the latitude and the lambāṁśa, or completement of the latitude. Or, the zenith distance and altitude of the Sun at mid-day when on the equinoctial give the latitude and its complement. (Wilkinson, 1861: 170)

The above verse expects the elevation of dhruva to be observed using an instrument (yantra) in order to find latitude. The dhruva is used in the context of finding direction in another context (in Tripraśnādhikāra 9a):

तन्मत्स्यादथ याम्यसौम्यककुभौ सौम्या ध्रुवे वा भवे–
देकस्मादपि भाग्रतो भुजमितां कोटीमितां शङ्कुतः ।
tanmatsyādatha yāmyasaumyakakubhau saumyā dhruve vā bhave-
dekasmādapi bhāgrato bhujamitāṃ koṭīmitāṃ śaṅkutaḥ.

From that fish, the northern and southern ends (directions) [are to be deduced]. Or, the northern [direction] (is fixed as being) at the dhruva. …

The relevant portion of this verse is "saumyā dhruve vā bhavet", i.e., the northern direction can be fixed using dhruva. The verse that precedes this one mentions a method by which a śaṅku (gnomon) is fixed on a flat surface and the eastern and western directions are fixed based on the shadows during the forenoon and afternoon of this gnomon. Two intersecting arcs are thought of as being fish-shaped, hence Bhāskara II says "from that fish". From east and west, one must derive north and south. The deduction is similar to that which is given by Lalla above.
Instead of using shadows to determine directions, Bhāskara II says that the location of dhruva can be fixed as north. The rest of the verse mentions a method to mark the directions using the position of dhruva, indicating once again that a certain degree of accuracy is expected.
Hence, Bhāskara II seems to have been acquainted with a visible pole star.

Padmanābha

Padmanābha mentions the following regarding the Dhruva in his Dhruvabhramādhikāra (11):

या तूत्तरा तारा सा पलांशैः क्षितिजादुपरितो वरीवर्ति । तत्परितो द्वादशतारकाभिर्मत्स्याकारमण्डलमुपलक्ष्यते । तस्य ध्रुवमत्स्यसंज्ञा विहिता । तन्मुखे पुच्छे स्थूले तारके द्वे दृश्येते । तयोर्मध्ये या मुखस्था सा ध्रुवतारायास्त्रिभिरंशैरन्तरिता । या पुच्छस्था सा तु त्रयोदशभिरंशैरन्तरिता वर्तते । उभे परस्परं षोडशभागान्तरिते स्तः ।
yā tūttarā tārā sā palāṁśaiḥ kṣitijāduparito varīvarti | tatparito dvādaśatārakābhirmatsyākāramaṇḍalam upalakṣyate | tasya dhruvamatsyasaṃjñā vihitā | tanmukhe pucche sthūle tārake dve dṛśyete | tayormadhye yā mukhasthā sā dhruvatārāyāstribhiraṁśairantaritā | yā

pucchasthā sā tu trayodaśabhiraṃśairantaritā vartate | ubhe parasparaṃ ṣoḍaśabhāgāntarite staḥ |

The northern Pole Star lies above the horizon at the degrees of the local latitude. Around the latter is seen a fish shaped constellation consisting of twelve stars. This is designated as the Polar Fish (dhruva matsya). Two bright stars are visible at its mouth and tail. Of these, the one at the mouth lies at an interval of three degrees (bhāga) from the [actual] Pole Star and the one at the tail lies at thirteen degrees. (Sarma, 2012: 325)

The above text is called Dhruvabhramaṇākhyaṭīkā by Durgaprasad Dvivedi in his Upapattīnduśekhara (1936) and the corresponding section has been quoted by him. This was translated by Shukla (1985: 596).

In a section prior to this, the text describes two dhruvas being established in the sky – one at the north pole and one at the south. Padmanābha uses the word 'sthūla' (large, big) to characterise these stars and additionally states that the fish's mouth is located 3° away from the true pole star (dhruvatārā in the quote above). The choice of words leads one to believe that he feels there is a faint star located at the pole itself.

Sarma (2012) identifies the two bright stars of dhruvamatsya as Polaris (α Ursae Minoris, at the mouth) and Kochab (β Ursae Minoris), also called Markaṭī in Sanskrit (at the tail) of the fish.

Parameśvara

Parameśvara (c. 1350 – 1450 CE) is one of the members of a long tradition of astronomers who lived in Kerala during the 14th-17th centuries, often known by the name the Kerala school of astronomy. It is known from Parameśvara's own writings and those of his successors that he observed the sky for about 55 years and introduced various changes based on his observations.

In the following verses, Parameśvara indicates the presence of a northern and a southern pole star (Goladīpikā 3.26-27):

मेरोरूर्ध्वगताग्रादूर्ध्वस्था तारका ध्रुवाख्यैका ।
मेरोरधोगताग्रादधः स्थितान्या ध्रुवाख्यतारा स्यात् ।।
ध्रुवयुग्ममिदं गोलस्याश्रयदण्डाग्रयुग्ममिव मेरौ ।
रशनावदतः परितो लङ्कायां तूर्ध्वगो भ्रमति गोलः ।।
merorūrdhvagatāgrādūrdhvasthā tārakā dhruvākhyaikā.
meroradhogatāgrādadhaḥ sthitānyā dhruvākhyatārā syāt.
dhruvayugmamidaṃ golasyāśrayadaṇḍāgrayugmamiva merau.
raśanāvadataḥ parito laṅkāyāṃ tūrdhvago bhramati golaḥ.

Above the upper extremity of Meru is a star named Dhruva (Pole-star, Polaris). Below the lower extremity of Meru must also be another star called Dhruva [South Pole-star]. These two Pole-Stars are, as it were, the extremities of the [central] axis supporting the [Celestial] sphere. Hence as seen from Meru the sphere [seems to] rotate around like a waist-band [of the Earth]; as seen from Laṅkā, however, it [seems to] rotate passing overhead [from east to west]. (Sarma, 1957: pp. 86-87)

The relation of the pole star with latitude is given in the following (Goladīpikā 3.8-11):

लङ्कायां क्षितिजस्थो ध्रुव उपरि स्वमध्यगः स मेरौ स्यात् ।
लङ्कायां विषुवस्थो दिनमध्ये व्योममध्यगः सूर्यः ।।
भानुः स एव मेरुस्थाने तु क्षितिजसंस्थितो भवति।
लङ्कात उदग्गमनाद्ध्रुवोन्नती रविनतिश्च कल्प्यातः ।।
सार्कावनतिर्देशे स्वेऽक्षज्या स्याद्ध्रुवोन्नतिर्वापि ।
अक्षज्यार्कावनतिध्रुवोन्नतीनां क्रमादुदग्वृद्धिः ।।
अक्षज्यावृद्धिवशादुत्तरगमनं ततस्तु लङ्कातः ।
ज्ञेयं तथा स्वदेशादक्षाधिक्येन सौम्यगमनमपि ।।
laṅkāyāṃ kṣitijastho dhruva upari svamadhyagaḥ sa merau syāt.
laṅkāyāṃ viṣuvastho dinamadhye vyomamadhyagaḥ sūryaḥ.
bhānuḥ sa eva merusthāne tu kṣitijasaṃsthito bhavati.
laṅkāta udaggamanāddhruvonnatī ravinatiśca kalpyātaḥ.
sārkāvanatirdeśe sve'kṣajyā syāddhruvonnatirvāpi.
akṣajyārkāvanatidhruvonnatīnāṃ kramādudagvṛddhiḥ.
akṣajyāvṛddhivaśāduttaragamanaṃ tatastu laṅkātaḥ.
jñeyaṃ tathā svadeśādakṣādhikyena saumyagamanamapi.

The Pole-star is [seen] at Laṅkā on the horizon [at the north point]; it will be right above at the centre of the heavens [as see] from Meru. The Sun on the Celestial Equator will be at the centre of the heavens at midday at Laṅkā. The same Sun will be on the horizon [as seen] from Meru. Hence as we go northwards from Laṅkā, we have to postulate the increase in the altitude of the Pole-star and the zenith distance of the midday Sun. This zenith distance of the Sun or altitude of the Pole-star will be equal to the latitude at the place [of observation]. The latitude, zenith distance of the Sun and altitude of the Pole-star gradually increase [as one proceeds] towards the north. Hence from an increase in the latitude, northward displacement from Laṅkā has to be inferred; and in the same manner, by an increase in latitude from one's own place, northward displacement from that place [can also be inferred]. (Sarma, 1957: pp. 83-84).

Nīlakaṇṭha

Nīlakaṇṭha Somayājī (c. 1444 – 1545 CE) was also one of the members of the Kerala school. He mentions the dhruva in Golasāra (2.4-5) as follows:

तत्र भ्रमन्ति यत्र क्रमेण दृश्यानि भानि सर्वाणि।
पार्श्वस्थे ध्रुवतारे निरक्षसंज्ञो भुवि प्रवेशः सः ।।
घटिकामण्डलमाहुस्तत्र यदधऊर्ध्वगं भ्रमद्वृत्तम्।
अभितोऽपि च तद्भ्रमतां भवन्ति नाना ध्रुवा द्युवृत्तानि।।
tatra bhramanti yatra krameṇa dṛśyāni bhāni sarvāṇi,
pārśvasthe dhruvatāre nirakṣasaṃjño bhuvi praveśaḥ saḥ.
ghaṭikāmaṇḍalamāhustatra yadadhaūrdhvagaṃ bhramad vṛttam,
abhito'pi ca tad bhramatāṃ bhavanti nānā dhruvā dyuvṛttāni.

That region upon the earth where all the stars revolving in that (Pravaha) region can be seen rising one after another, and where the Pole stars are (exactly) on the two sides, is called the region of Zero latitude (i.e., the Equatorial region). The revolving vertical circle there (i.e., the Prime Vertical at that region) is called the Ghaṭikā-maṇḍala (Hour Circle). On both sides of that are the different fixed Diurnal Circles of the heavenly bodies. (Sarma, 1970: 8)

He uses the word 'dhruvatāre' (dual of dhruvatārā).

Jyeṣṭhadeva

Jyeṣṭhadeva (c. 1500 – 1610 CE) was a junior contemporary of Nīlakaṇṭha and the author of Yuktibhāṣā, a treatise in Malayalam, that discusses mathematics and astronomy. The reference pertaining to dhruva is given below:

āvṛttamārggattiṅkal ellāṭattinnuṁ dhruvan enna oru nakṣatratte bhūpārśvattiṅkal tekkuṁ vaṭakkuṁ anudayāstaṁ kāṇāṁ. ī pradēśattiṅkannu vaṭakkōṭṭu nīṅṅiyāl vaṭakke dhruvanē kāṇāvū. vaṭakku nīṅṅiyōḷaṁ uyarnnirikkuṁ ī dhruvan. ī dhruvōnnatiye akṣaṁ ennu collunnū. (Chapter 9, p. 899)

From all places on that (equatorial) line, can be seen two nakṣatra-s (stars) called Dhruva-s (pole stars), one in the north and one in the south, which have no rising or setting. If one moves to the north from this line one can see only the northern Dhruva. This Dhruva would have as much altitude as one moves towards the north. This altitude of the Dhruva is called akṣa (the terrestrial latitude). (Sarma *et al*., 2008: 510)

It is observed here that Jyeṣṭhadeva essentially defines latitude based on the elevation of the pole star.

Nityānanda

Nityānanda (c. 1630 CE), who was the court astronomer of the Mughal Emperor Shah Jahan, mentions the following in his Gola (verse 29)

निरक्षदेशाच्चलितः पुमान् यथा कुबेरकाष्ठां च तथोन्नतं ध्रुवम् ।
विलोकयेन्नम्रमयीह वैषवं नभस्तलाद्याम्यविभागसंस्थितम् ।।

nirakṣadeśāccalitaḥ pumān yathā kuberakāṣṭhāṃ ca tathonnataṃ dhruvam,
vilokayennamramayīha vaiṣavaṃ nabhastalādyāmyavibhāgasaṃsthitam.

To the extent that a man moves from the equatorial region (nirakṣadeśa) and to the northern region, by that amount [he] should regard the pole star (dhruva) elevated (unnataṃ) [above the northern horizon]. He should regard the celestial equator (viṣuvavṛtta) going downwards from the zenith (nabhastala) directed towards the southern part [with equal relative displacement]. (Misra, 2016: 153)

Misra (2016) interprets this star as Polaris.

Discussion

From the above mentions, it is clear that references to dhruva are of several kinds. It would thus be useful to classify their approach and understand those which refer directly to the pole star.

Classification of References

From the mentions of dhruva quoted above, it is found that references to dhruva are for primarily three purposes:

- Latitude: the elevation of the pole star indicates the latitude
- Direction: the pole star and pole in general is the northern direction
- Cosmology: that there are two poles, fixed in the north and south, around which the sky appears to rotate.

Looking at these in another way, in their allusion to the pole, these references are of two kinds:

1. Ambiguous: these can be interpreted as pertaining either to the north pole or the pole star, e.g. Lalla who states that dhruva indicates the northern direction.
2. Definitive: these state explicitly that the object under question is a star, e.g. Vaṭeśvara's dhruvatārā.

Those references which expect measurement or observation of some kind so as to arrive at latitude, but do not explicitly use the word tārā (e.g. as stated by Śrīpati), should be considered as referring to a star, as it is obviously impossible to make a measurement with reference to the empty night sky. Therefore, definitive references (where a star is clearly stated or implied) are of the following types with increasing degree of precision:

1. Cosmological: that two stars are present in the north and south celestial poles respectively (e.g. the Kerala School astronomers)
2. Measurement: when the author mentions the procedure to measure the latitude using the dhruva (a star may be explicitly mentioned or implied, e.g. Varāhamihira)
3. Fuzzy location references: where the dhruvatārā is mentioned as being roughly located in an area (e.g. Vaṭeśvara)
4. Precise location references: where the location is mentioned with a measure of degrees.

One may assume that latitude will have to be known to a certain degree of precision in order to arrive at astronomical parameters correctly (e.g. in Bhāskara's observation with a yantra), hence, those authors who find the latitude based on dhruva would have preferred a star closer to the pole than one farther away. This would indicate a higher degree of precision being expected. However, this is an assumption and accuracy of the parameters stated in individual texts would have to be examined as a separate study.

Out of the four categories in definitive references mentioned above, the cosmological references may not be of particular benefit for pinpointing a star, since this may represent a world view, rather than an empirical statement. However, the other three categories hold information that is useful in the present study.

Although the references are not always precise, as seen above, there is an indication of one or more stars. This star may have been close to or somewhat displaced from the pole, it may be bright or faint, however, it is clear that some visible star is intended. Given that most authors in the Siddhāntic period have references that fall in these categories, it appears that one or more stars had the identity of 'dhruvatārā' during this period. It is therefore worthwhile to examine the references above and the available evidence to bring together information about the nature and possible identity(ies) of this star.

In Focus

Some specific references will be considered below:

1. Vaṭeśvara

Vaṭeśvara mentions the pole star as being a rather faint one (tanutarā) somewhere in the middle or within (madhye) the polar fish (timyākṛtitārāṇām). The polar fish (dhruvamatsya) is discussed in many other texts as seen from the quotes above. The exact identity of this constellation is not known; however, it is evident from the name that it comprised of stars somewhere around the pole.

Vaṭeśvara also states that the pole star is fainter or "rather faint" (tanutarā). This could indicate that the star is faint in comparison to brighter objects such as planets (i.e., comparing across the sky), or that it is faint in comparison to the stars in its neighbourhood.

In the first interpretation, this can be compared to the brightness of dimmer visible planets: the mean magnitude of Mercury is about 0.1 and that of Saturn is 0.5; their faintest magnitudes are 5.6 and 1.2 respectively (Mallama & Hilton, 2018). Any star in the range of perhaps 2.0 to 5.5 would count

as faint. This includes Polaris, whose apparent visual magnitude is around 2.0, and most other stars that are in the polar region.
In the second interpretation, the range occupies a much lower level on the scale. Polaris, one of the brightest stars in the region has magnitude 2.0. Thuban, which is believed to be the pole star of the Vedas, has a magnitude of about 3.7.
In both cases, the lower bound of magnitude must be 6.5, since in a clear sky that is devoid of light pollution, the average unaided eye is only capable of viewing objects that have magnitude up to 6.5 (Curtis, 1903).
Thus, Vaṭeśvara probably referred to a faint star located close to where the pole was during his time.

2. Padmanābha

Padmanābha refers to the polar fish and then states the distance (in degrees) of mouth and tail of the fish from the pole itself. These stars have been identified as Polaris and Kochab, and this can be verified from the distances he has given for his epoch.
However, Padmanābha also refers to a "pole star" distinct from this, calling the other stars 'sthūla'. This would mean that the star in question is fainter than Polaris and Kochab (both having magnitude ~2.0).
As the separation from the pole of Polaris and Kochab have been given in degrees, we can assume that the pole star is within a degree of the actual pole, hence it has been named as such. This would put this reference in the 'Precise location reference' category.

Stars within a Degree

Based on this, a list of stars that approached within 1° of the celestial pole during the Siddhāntic period, has been compiled in Table 1. As is evident, although close to the pole, most of the stars are very faint and only barely visible. The brightest before Polaris is Σ1694, which was close to the pole during most of the period under consideration.

Table 1: List of visible pole stars after 1 CE with year of closest approach, declination in this year, and magnitude (from Stellarium).

Star name / designation	Year of closest approach	Declination in this year	Magnitude
HIP 62170	314 CE	89° 58′	6.30
HIP 59384	553 CE	89° 3′	6
Σ1694	806 CE	89° 27′	5.3
HIP 58874	1207 CE	89° 43′	6.25
HIP 59879	1358 CE	89° 59′	6.3
HIP 59767	1587 CE	89° 57′	6.25
Polaris	2100 CE	89° 33′	1.95

The bound of 1° has been assumed keeping in mind references like that of Padmanābha, and considering that the latitude was known to the nearest degree. However, there is scope to expand these boundaries and consider other stars that can approach close to pole. The possibilities indicated on expanding this bound, and their viability would have to be undertaken as a separate study.
It is unlikely in case of most references that Polaris could have been the intended star. This is because at Bhāskara II's time, it was about 5.4° from the pole, this value being greater before his time. Further, the statement of Vaṭeśvara about the star being 'tanutarā' is unlikely to match with Polaris. In case of Padmanābha, as he has stated that the star is different from Polaris, hence the question does not arise. In case of the later authors, there is scope for doubt.

During the period 600 – 1000 CE, the brightest star among those that were visible and close to the pole, as seen above, was Σ1694. This star is still faint, and fits the adjective of 'tanutarā' that Vaṭeśvara ascribes to the pole star of his time. HIP 59384 was closer during Varāhamihira's time by about half a degree than Σ1694. HIP 59879 is the closest during Padmanābha's period. After 1500 CE, Polaris emerges as the primary candidate.
It is possible that knowledge of the exact location and identity of the pole star was passed down through the guru-śiṣya paramparā (i.e., from teacher to student), as an oral tradition. The identity of dhruva may have also been different in different places and time periods. A more detailed analysis into the available literature may shed more light on this. However, with the available information, some possibilities have been explored.

Conclusion

The search for the pole star in the Siddhāntic literature is indeed fascinating. Several texts across centuries mention it even though, as we know, the pole keeps moving, and several stars have occupied the position of the pole star. This is in contrast to the prevalent opinion that the siddhāntas are not acquainted with a pole star.
The texts considered here date from the sixth to seventeenth centuries CE. Various mentions of the pole star in Indian astronomical literature during this time period have been brought together.
It is observed that dhruva is often used to measure latitude (through the elevation of the pole) which cannot be accomplished without a visible star (or object) at the pole. Although dhruva often connotes the north celestial pole, the use of the word dhruvatārā in several texts and the practical use of dhruva for latitude measurement, seem to indicate that a visible star would have been observed close to the pole.
Considering the different statements and the locations of the stars in the sky during the period under consideration, the preliminary study conducted here identifies Σ1694 as a likely candidate during the period 600 – 1000 CE, and various stars during other time. However, there is scope for further study and more evidence needs to be gathered for arriving at a proper conclusion. We would like to call the attention of researches to this matter.

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